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EFFICIENT ALGORITHMS FOR THE EVALUATION OF PLANAR NETWORK RELIABILITY

FINAL REPORT

By

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13. ABSTRACT (*Maximum 200 words*)

Computation of the all-terminal reliability for general classes of networks is NP-hard. The problem of characterizing subclasses of networks and designing efficient algorithms to compute their reliability is a major area of current research in network reliability. Supported by the ARO grant DAAL03-90-G-0078, we have made significant progress in enlarging the class of planar networks for which there exists efficient all-terminal reliability algorithms. The importance of this work is enhanced by the recent proof of Vertigan that the all-terminal network reliability problem for the full class of planar networks is NP-hard. A important outgrowth of our work is the discovery of general techniques for developing $O(n \log n)$ algorithms for a greatly enlarged class of planar and nonplanar networks.

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1. FOREWORD

Reliability is one of the primary considerations in the design of communication systems, computer systems and power transmission systems. Such systems are modelled as graphs or digraphs whose elements (vertices and/or edges) have an associated probability of being operational, and the reliability considerations fall into the area known as *network reliability*. A key issue in network reliability is the determination of the reliability of a given network system from the reliabilities of its components. Procedures for computing network reliability date back to the 1950's; however, due to increasing network complexity, considerable attention has been given recently to the exploration and discovery of efficient computational methods, resulting in several outstanding advances in network reliability. Particular attention has been paid to the *all-terminal reliability* R(G) of a graph whose edges fail with known probabilities, where R(G) is the probability that G is connected.

Computing the all-terminal reliability for a general network has been shown to be an NP-hard problem [14,17], and thus it is unlikely that an efficient polynomial running time procedure can be constructed for solving general networks. Very recently, it was also established that even for planar networks the computation of R(G) is NP-hard [18]. In work supported by the two year ARO grant, we have studied special classes of networks and as a result developed efficient algorithms for progressively larger subclasses of the class of planar networks. This ARO supported effort involved two PhD students and resulted in about 10 refereed journal papers.

An important contribution of the work is the discovery of a general technique to obtain efficient algorithms for certain important classes networks by the use of *forbidden minor characterizations* of these networks. While forbidden minor characterizations have a long and rich history, there seems to have been no known connection between such characterizations of graphs and their reliability analysis. Our research centered on forbidden minor characterizations themselves and their subsequent application to the design of efficient reliability algorithms.

2. THE REPORT

A. STATEMENT AND AN OVERVIEW OF THE NETWORK RELIABILITY PROBLEM

In this section we present an overview of the known classes of networks whose reliability can be computed efficiently. First, we need the description of the K-terminal reliability measure and the notions of *replacements* and *reliability preserving network reductions.*

Suppose $G = (V, E)$ is a graph and $K \subseteq V$ is a specified subset of the vertex set V. Furthermore, the elements (vertices and/or edges) of G may fail with known probabilities. The K-*terminal reliability* $R_K(G)$ is the probability that there is a path of operating elements between every pair of vertices in K. The all-terminal reliability $R(G) = R_V(G)$ is a special case of the K-terminal measure.

Reliability Preserving Reductions:

In order to reduce the size of G and therefore reduce the complexity of computing $R_K(G)$, *reliability preserving reductions* are often applied. For each edge e_a, we will denote the probability that e_a operates by p_a and the probability that e_a fails by q_a. The following reliability preserving reductions are well known.

Parallel reduction: Suppose that $e_x = uv$ and $e_y = uv$ are two parallel edges in a graph G. Then a parallel reduction replaces e_x and e_y with a single edge $e_z = uv$ such that $p_z = 1 - q_xq_y$.

Series reduction: Suppose that $e_x = uv$ and $e_y = vw$ are two edges such that v is a degree 2 point in graph G and v is not a K-point. Then a series reduction replaces e_x, e_y and the vertex v with a single edge $e_z = uw$ such that $p_z = p_xp_y$.

If G′ is the graph obtained from G after a series or parallel reduction, then $R_K(G) = R_K(G')$.

Degree-2 reduction: Suppose that $e_x = uv$ and $e_y = vw$ are two edges such that v is a degree 2 point in G and $\{u, v, w\} \subseteq K$. Then a degree-2 reduction replaces e_x and e_y with a single edge $e_z = uw$ such that $p_z = p_x p_y/(1 - q_x q_y)$. If G′ is the graph obtained from G after a degree-2 reduction, then $R_K(G) = (1 - q_x q_y)R_{K-v}(G')$.

Degree-1 reduction: Suppose that $e_x = uv$ such that u is a degree 1 point in G and $u \in K$. Then a degree-1 reduction deletes point u and edge e_x. If G′ is the graph obtained from G after a degree-1 reduction, then $R_K(G) = p_u R_{(K \cup v)-u}(G')$ if $u \in K$ and $R_K(G) = R_K(G')$ otherwise.

Loop reduction: Suppose that edge e = uu is a loop of graph G. Then a loop reduction deletes edge e from G. If G′ is the graph obtained from G after a loop reduction, then $R_K(G) = R_K(G')$.

Suppose {a, b, c} *is a minimal point disconnecting set* of a connected graph G. Let H′ be a connected component of G - {a, b, c}. The subgraph, say H, induced by $V(H') \cup \{a, b, c\}$ is called a *3-attached subgraph* of G. The points a, b, and c are the *attachments* of H, while the points in V(H′) are the *internal points* of H. The points V(G) - V(H) are *external points* of H. Note that every 3-attached subgraph has at least one internal point and at least one external point. A 3-attached subgraph with two or more internal points is a *non-trivial 3-attached subgraph*. Suppose H is a non-trivial 3-attached subgraph of G. If H has no proper subgraph which again is a non-trivial 3-attached subgraph of G, then H is called a *trisubgraph* of G. In other words, a trisubgraph is a minimal non-trivial 3-attached subgraph.

Trisubgraph to Y reduction: Suppose that H is a trisubgraph of G with attachments {a, b, c} and $\{a, b, c\} \subseteq K$. A trisubgraph-to-Y reduction deletes the internal points of H and any existing edges between {a, b, c}, adds a non-K point w and edges x = wa, y = wb, z = wc. The probabilities of w, x, y and z are as follows:

$p_x = \alpha/(\alpha + \beta_{a,\{b,c\}})$, $p_y = \alpha/(\alpha + \beta_{b,\{a,c\}})$, $p_z = \alpha/(\alpha + \beta_{c,\{a,b\}})$ and $p_w = A/(A + B)$.

Moreover, A, B, α, $\beta_{a,\{b,c\}}$, $\beta_{b,\{a,c\}}$, $\beta_{c,\{a,b\}}$ and γ are given by:

A $= (\alpha + \beta_{a,\{b,c\}})(\alpha + \beta_{b,\{a,c\}})(\alpha + \beta_{c,\{a,b\}})$.

B $= \alpha^2\gamma - (\alpha\beta_{a,\{b,c\}}\beta_{b,\{a,c\}} + \alpha\beta_{b,\{a,c\}}\beta_{c,\{a,b\}} + \alpha\beta_{a,\{b,c\}}\beta_{c,\{a,b\}} + \beta_{a,\{b,c\}}\beta_{b,\{a,c\}}\beta_{c,\{a,b\}})$.

α : the probability that every K-point in H is connected to every point of {a, b, c}.

$\beta_{i,T}$: the probability that every K-point in H is connected to either i or to all points of T, but not both, where $i \in \{a, b, c\}$ and $T = \{a, b, c\} - \{i\}$.

γ : the probability that every K-point in H is connected to exactly one of {a, b, c}.

If G′ is the graph obtained from G after a trisubgraph to Y reduction, then $R_K(G) = ((A + B)/\alpha^2)\, R_{K-I(H)}(G')$, where I(H) is the set of internal points of H.

Graphs with cutpoints:

A connected graph is said to be *separable* if it has a cutpoint. Let G = (V,E) be a separable graph. If $u \in V$ is a cut-point of G, then we can decompose G into two subgraphs $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ such that $G = G_1 \cup G_2$, and $G_1 \cap G_2 = \{u\}$. It is well-known that $R(G) = R(G_1)R(G_2)$. This decomposition can be repeated on subgraphs G_1 and G_2, and so on, until all the subgraphs obtained are nonseparable.

Note that the nonseparable components of G can be found in time $O(|V|+|E|)$ using the Depth-First-Search. Furthermore it is clear that the reductions, series, parallel, degree-2, and degree-1 are constant time reductions. For this reason we need only consider nonseparable graphs which admit none of these four reductions.

Graphs with separation-pairs:

A pair of points u and v in a nonseparable graph G is a *separation-pair* if the deletion of u and v from G results in a disconnected graph. Clearly a nonseparable graph with no separation pairs is 3-connected. Suppose that (u, v) is a separation pair in a nonseparable graph G, then we can decompose G into two subgraphs $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ such that $G = G_1 \cup G_2$, $G_1 \cap G_2 = \{u, v\}$, and $E_1 \cap E_2 = \phi$. It is known that, for the case of all-terminal reliability, any of these subgraphs can be replaced by a single edge to yield a reliability preserving reduction. Specifically, if G′ is the graph obtained from G_1 by adding an edge $e_a = \{u, v\}$ to G_1, then $R(G) = \sigma\, R(G')$; the values of σ and p_a depend on $R(G_2)$. This decomposition can be repeated on graphs $G_1 + e_a$ and $G_2 + e_a$, and so on, until all the graphs obtained are 3-connected.

The 3-connected components of G can be obtained in time $O(|V|+|E|)$ using the algorithm of Hopcroft and Tarjan [7]. Thus we need only consider 3-connected graphs.

Graph replacements:

Consider the following six operations (replacements) on graphs without regard to probabilities:

(i) Parallel replacement : deletes an edge that is parallel to another edge

(ii) Series replacement: contracts an edge that is in series with another edge

(iii) Degree-1 replacement: deletes a degree one vertex

(iv) Loop replacement: deletes an edge that is a loop

(v) Y-Δ replacement: deletes a degree 3 vertex, say u, whose neighbors are {a, b, c} and adds new edges between the pairs {a, b}, {b, c} and {a, c}

(vi) Δ-Y replacement: the inverse of Y-Δ replacement.

Replacements are operations involving only replacement of some edges or vertices of a graph G by other edges or vertices to obtain a new graph G′. A reduction, on the other hand, is defined with respect to G, K, and edge reliabilities. A reduction in G includes the act of replacement to create G′ along with defining edge reliabilities, K′ and Ω, all such that $R_K(G) = \Omega R_{K'}(G')$, i.e. reliability is preserved. To underpin this distinction in network reliability algorithms, let us first consider the well-known class of series-parallel graphs.

Series-parallel graphs

A graph is a *series-parallel graph* if it can be reduced to an edgeless graph by a finite sequence of the operations (i) thru (iv). This definition should not be confused with the definition of a "two-terminal" series parallel network in which two vertices must remain fixed. No special vertices are distinguished here. For example, it is easy to see that the graph of Fig. 1 is a series-parallel graph. In particular, one may apply reliability preserving reductions for the network shown in Fig. 1a to reduce it to an edgeless graph; i.e., the reliability of the network of Fig. 1a is computable using a sequence of series, parallel and degree-1 reductions. However, if the K-vertices are different, as shown in Fig. 1b, the network does not admit any series, parallel or degree-1 reductions. Motivated by the difference between graphs which allow replacements but, with K and edge reliabilities defined, do not allow reliability preserving reductions, one can classify the series-parallel graphs into two types as follows:

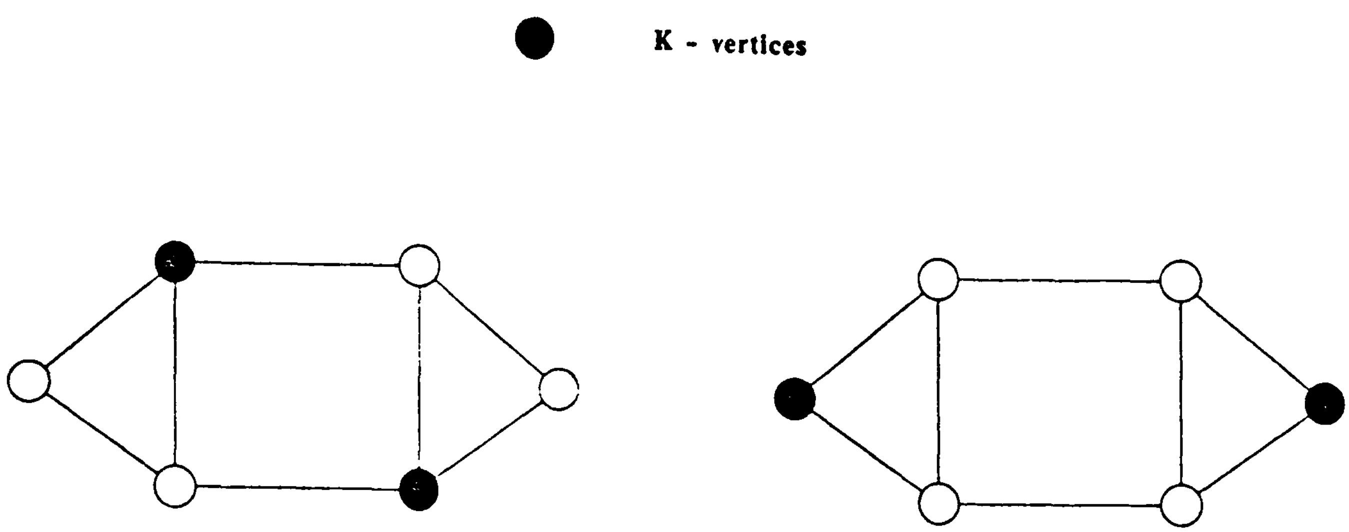


FIG. 1a. sp-reducible graph.

FIG. 1b. sp-irreducible graph.

Let G = (V,E) be a series-parallel graph such that the edges of G operate with known probabilities. For a given $K \subseteq V$, the graph G is said to be *s-p reducible* if it can be reduced to an edgeless graph by a sequence of series, parallel, degree-1 and degree-2 reductions. The series-parallel graph of Fig .1a is s-p reducible while that of Fig. 1b is *s-p irreducible.*

It is evident that the K-terminal reliability of an sp-reducible network G = (V,E) can be computed in O(|E|) time using series, parallel, degree-1 and degree-2 reductions. However, these reductions are not sufficient to compute the reliability of sp-irreducible networks. Satyanarayana and Wood [16] constructed a linear time algorithm to compute $R_K(G)$ of sp-irreducible networks, using a new reduction called *polygon-to-chain reduction.* A cycle C in a graph G is called a polygon if C has exactly two points of degree > 2 and every other point of C has degree equal to 2 in G. Satyanarayana and Wood showed that every sp-irreducible graph has a polygon which can be replaced by a chain such that the reliability of the network is preserved and the resulting graph is again a series-parallel graph. Similar results have also been obtained for directed networks. A directed graph is said to be *basically series-parallel* (BSP-digraph) if the underlying undirected graph is a series-parallel graph. An O(|E|) time algorithm to compute the source-to-K-terminal reliability of BSP-digraphs has been found by Agrawal and Satyanarayana [1,2].

Graph characterizations in terms of forbidden minors

If x = uv is an edge of G with the end points u and v, then by the *contraction of edge* x, we mean deleting x and identifying the points u and v to a single point. A graph H is *contractible* to a graph G if G is obtained from H by a sequence of edge contractions. A graph H is a *minor* of G if G has a subgraph contractible to H. Characterizations of certain classes of graphs in terms of some forbidden minors have a rich history in graph theory. For example, the classical result of Kuratowski may be stated as follows. A graph G is planar if and only if the complete graph K_5 or the complete bipartite graph $K_{3,3}$ is not a minor of G.

The well-known series-parallel graph characterization of Duffin [4] can be stated as follows. A graph G is series-parallel if and only if the complete graph K_4 is not a minor of G.

In the following discussion, we say that a graph G is a $(H_1, H_2, ..., H_m)$-*free* graph if graphs $H_1, H_2, ..., H_m$ are not minors of G.

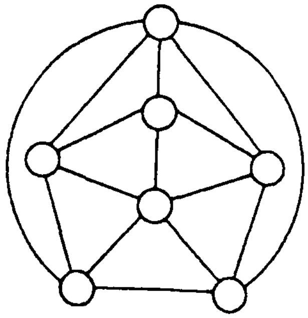
Y-Δ graphs, Δ-Y graphs and Δ-Y-Δ graphs

A graph is a Δ-Y-Δ graph if it can be reduced to an edgeless graph by a finite sequence of the replacements (i) thru (vi). A Δ-Y-Δ graph is a Y-Δ graph (Δ-Y graph) if the reduction sequence does not use the Δ-Y replacement (Y-Δ replacement). Δ-Y-Δ graphs have been known for a number of decades; series-parallel, Δ-Y and Y-Δ replacements have been used to simplify the analysis of electrical networks. Δ-Y-Δ graphs constitute a large class of graphs; indeed, Epifanov [5] showed that every planar graph is a Δ-Y-Δ graph.

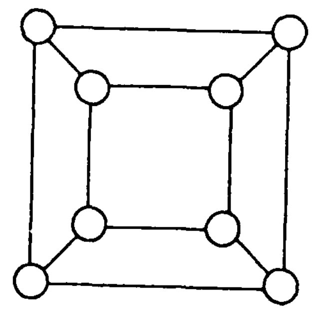
The recent forbidden minor characterizations of Δ-Y graphs and Y-Δ graphs led us to discover efficient reliability computation algorithms for these classes.

Δ-Y graphs:

Politof [9], in his PhD thesis, provided the first structural characterization of Δ-Y graphs. He showed that every Δ-Y graph is planar and that a planar graph is Δ-Y graph if and only if it is a $(C_5+2K_1, K_2 \times C_4)$-free graph. The graphs C_5+2K_1 and $K_2 \times C_4$ are shown in Fig. 2. This characterization turns out to be very useful in reliability analysis. Indeed, this yields an efficient algorithm to compute the reliability of a slightly larger class, namely the class of $K_2 \times C_4$-free graphs that include Δ-Y graphs. Politof and Satyanarayana [10,11] used this structural characterization and showed that R(G) of a $K_2 \times C_4$-free graph G can be computed in linear-time. Note that one could also compute R(G) of a Δ-Y graph using reliability preserving reductions discussed above, but this would result in an algorithm of complexity at least $O(n^2)$.



$C_5 + 2K_1$



$K_2 \times C_4$

Fig. 2 The graphs C_5+$2K_1$ and $K_2 \times C_4$

Y-Δ graphs:

Satyanarayana ana Tung [15] and Arnborg, Proskurowski and Corneil [3] independently showed that a graph is a Y-Δ graph if and only if it is a (K_5, $K_{2,2,2}$, $C_8(1,4)$, $K_2 \times C_5$)-free graph. Graphs K_5, $K_{2,2,2}$, $C_8(1,4)$ and $K_2 \times C_5$ are shown in Fig. 3. This characterization is again useful in reliability analysis and it yields an efficient algorithm to compute the reliability of a much larger class, namely the class of (K_5, $K_{2,2,2}$)-free graphs that include Y-Δ graphs. Politof and Satyanarayana and Tung [13] used the characterization and showed that R(G) of a (K_5, $K_{2,2,2}$)-free graph G can be computed in $O(n \log n)$, where n is the number of vertices of G. It is important to note that there is no Y-Δ reliability preserving reduction. However, using the forbidden minor characterization Y-Δ graphs it is possible to show that every Y-Δ graph admits a trisubgraph-to-Y reduction and the resulting graph is again a Y-Δ graph. This indeed is the basis of the $O(n \log n)$ algorithm of [13].

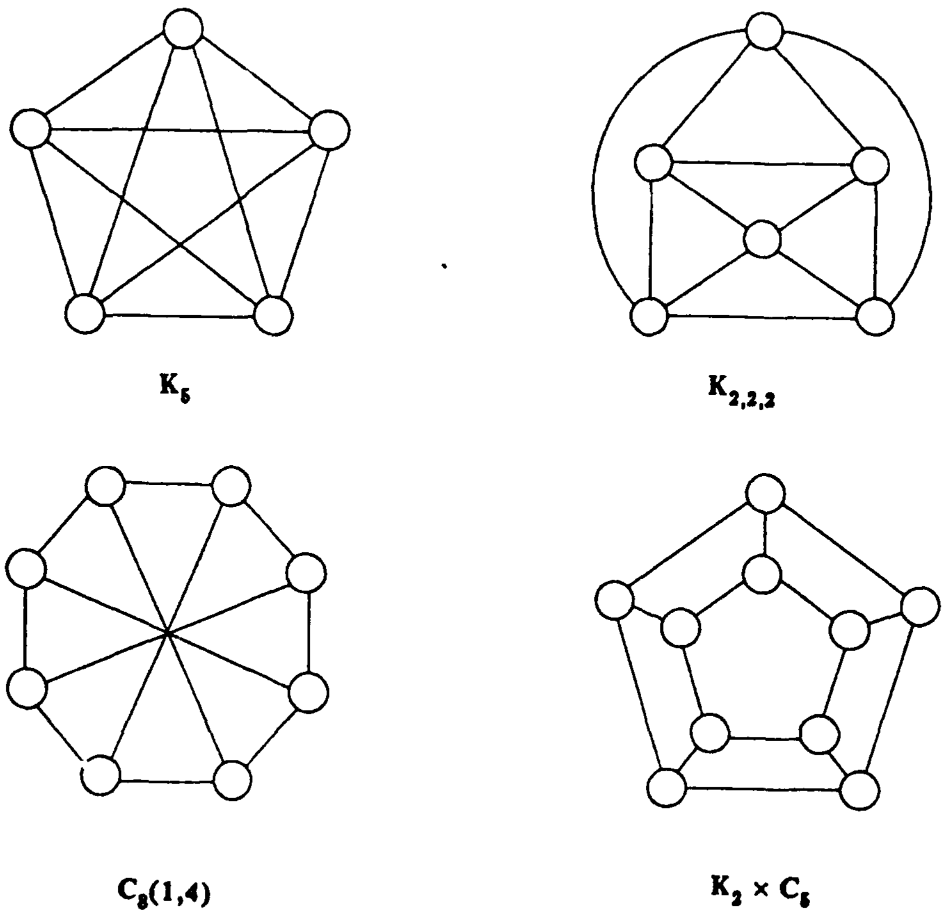


Fig. 3 The graphs K_5, $K_{2,2,2}$, $C_8(1,4)$ and $K_2 \times C_5$

B. SUMMARY OF RESULTS

An important outgrowth of the work during the second year of the grant period is the development of the notion of *quasi 4-connected graphs* and their characterization by Politof and Satyanarayana [12]. A minimal vertex disconnecting set S of a graph G is a *nontrivial* |S|-*separator* if the connected components of G-S can be partitioned into two sets each of which has at least two vertices. A 3-connected graph is quasi 4-connected if it has no nontrivial 3-separators. A super set of quasi 4-connected graphs, called Vosperian graphs, have been recently studied by Hamidoune and Tindell [6] from a group-theoretic point of view.

Politof and Satyanarayana [12] proved the following:

Theorem 1: Suppose G is a nonplanar quasi 4-connected graph. If K_5 is not a minor of G then G is either $C_8(1,4)$, or G has exactly six points and has $K_{3,3}$ as a subgraph.

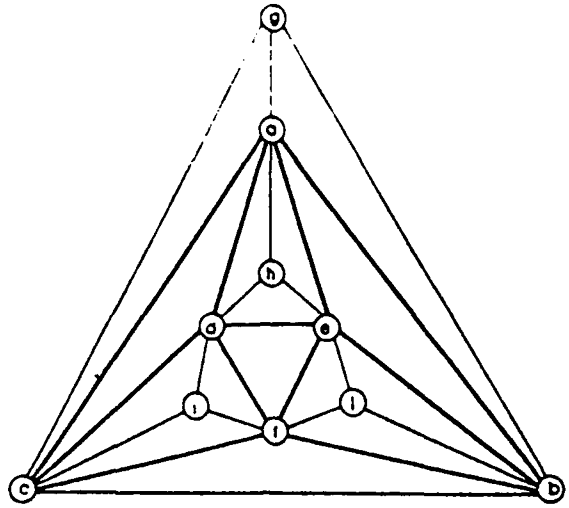
Theorem 2: Suppose G is a planar quasi 4-connected graph. If $K_2 \times C_4$ is not a minor of G and $G \neq C_5 + 2K_1$ then G is a subgraph of either P or Q. (Graphs P and Q are shown in Fig. 4).

Theorem 3: Suppose G is a planar quasi 4-connected graph. If $K_{2,2,2}$ is not a minor of G and $G \neq K_2 \times C_5$ then G is a subgraph of Q.

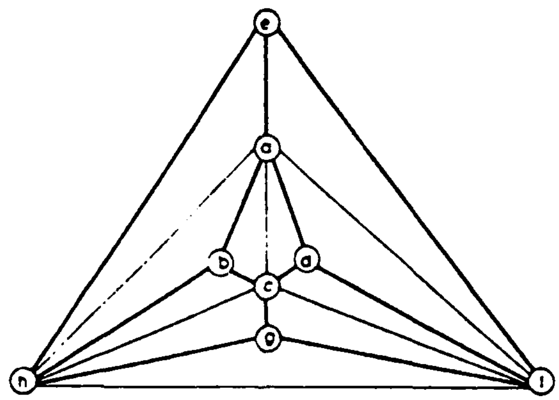
These results in conjunction with the known forbidden minor characterizations of Δ-Y and Y-Δ graphs yield the following propositions that form the basis for the construction of efficient algorithms for computing R(G).

Since a Δ-Y graph is a planar and $(K_2 \times C_4, C_5 + 2K_1)$-free graph, Proposition 1 is immediate from Theorem 2.

Proposition 1: Every 3-connected Δ-Y graph has a trisubgraph H which is a subgraph of graph P or Q.



The graph P



The graph Q

Fig. 4. Graphs P and Q

Since a Y-Δ graph is a (K_5, $K_{2,2,2}$, $C_8(1,4)$, $K_2 \times C_5$)-free graph, Proposition 2 is immediate from Theorems 1 and 3.

Proposition 2: Every 3-connected Y-Δ graph has a trisubgraph H such that H is one of the four graphs shown in Fig. 5 in which the broken lines correspond to edges that may or may not exist.

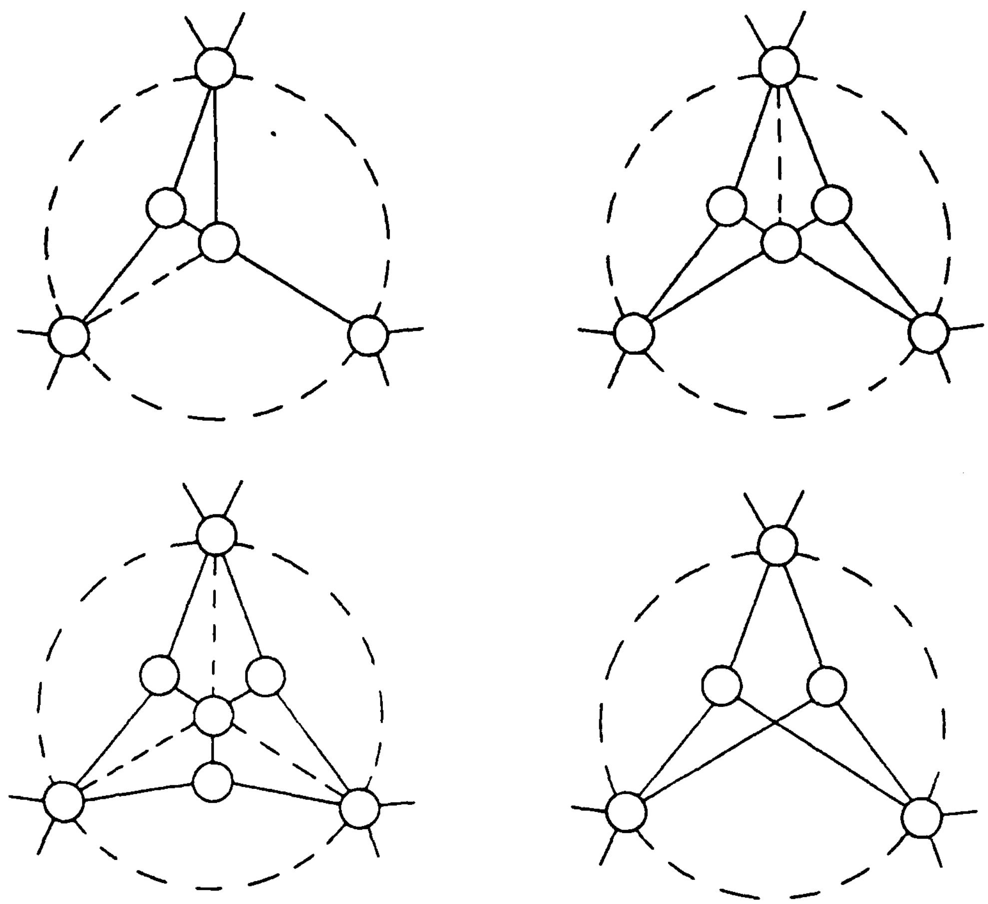


Fig. 5. Trisubgraphs of Y-Δ graphs

In both the cases, the propositions assert that there is a trisubgraph H in the given graph such that the number of points in H is bounded by a constant. Thus the trisubgraph-to-Y reduction can be executed in constant time and the resulting graphs belong to the same class again so that the argument can be repeated. Moreover, given a vertex, verifying whether it is an internal vertex of a trisubgraph can be done in constant time for Δ-Y graphs. It takes little more to show that $\mathcal{O}(\log n)$ time suffices to do the same in the case of Y-Δ graphs. Thus $\mathcal{O}(n)$ and $\mathcal{O}(n \log n)$ algorithms can be constructed to compute R(G) of Δ-Y graphs and Y-Δ graphs respectively.

The results and the discussion above lead to a general scheme to develop efficient algorithms for computing the all-terminal reliability of much wider classes of graphs whenever they satisfy the following property.

Let Γ be the collection of all (H_1, H_2, ..., H_k)-free graphs for some finite set of graphs {H_1, H_2, ..., H_k}. Let Γ_q be the collection of all quasi 4-connected graphs in Γ. Note that, in general, Γ is infinite. If the reliability of the graphs in Γ_q is efficiently computable, then the reliability of every graph in Γ is efficiently computable. In particular, this will be the case if Γ_q is finite.

G. Lingner, T.Politof and A. Satyanarayana [8] studied the following class of graphs and proved some important results. Consider the seven graphs shown in Fig. 6. We say that a graph G belongs to $\mathcal{P}$ if none of the seven graphs of Fig. 6 is a minor of G. They proved that there are finitely many quasi 4-connected graphs in $\mathcal{P}$ and moreover that each such graph has at most 14 vertices. From this it is shown that R(G) of any G in $\mathcal{P}$ is computable in $\mathcal{O}(n \log n)$ time where n is the number of vertices of G. The class $\mathcal{P}$ contains both planar and nonplanar graphs. In fact $\mathcal{P}$ contains all Δ-Y graphs and Y-Δ graphs.

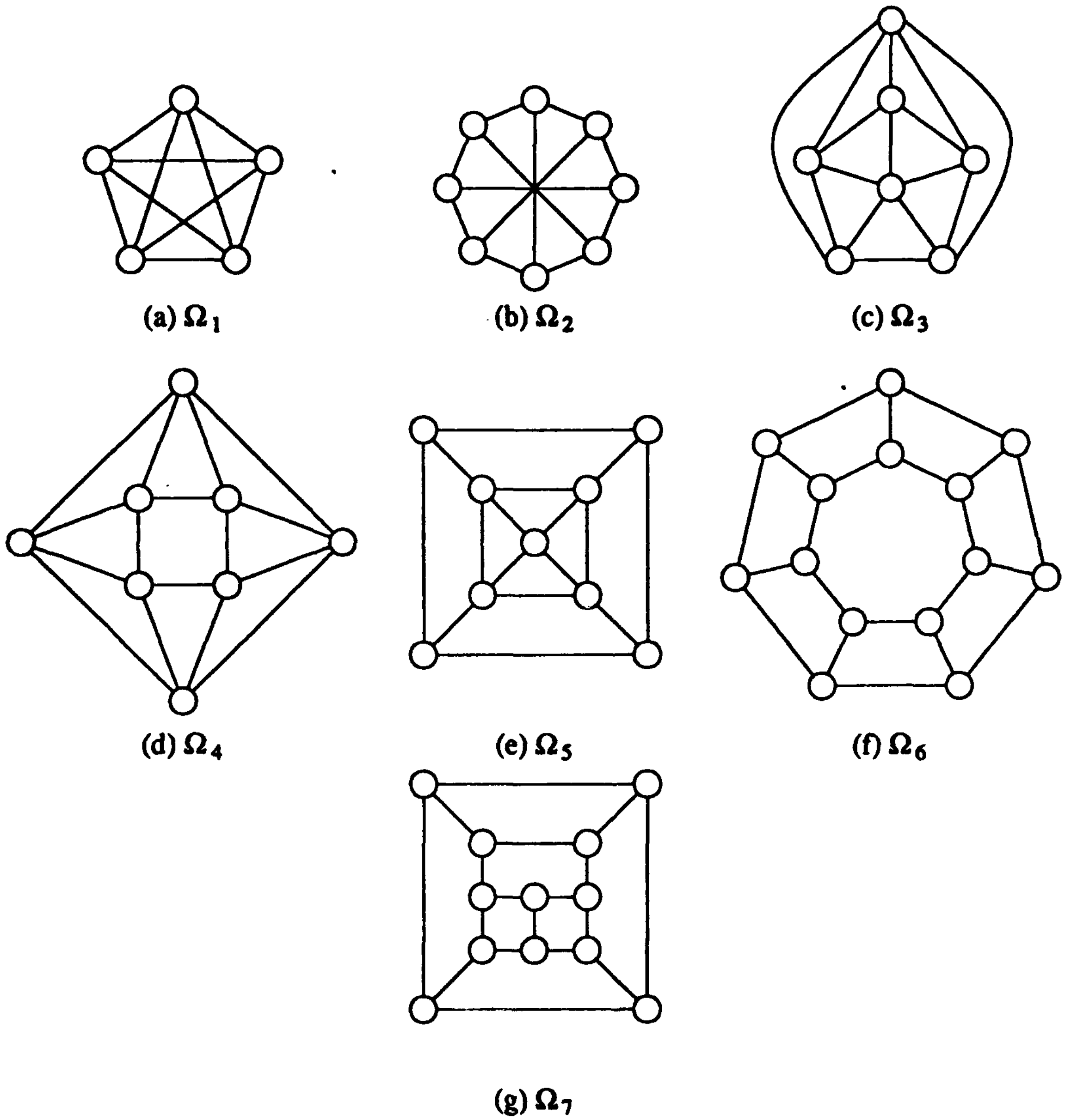


Fig. 6. The Seven Forbidden Graphs

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